

SPECTRAL FUNCTION AND KINETIC EQUATION FOR NORMAL FERMİ LIQUID

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ABSTRACT. On the basis of the Kadanoff-Baym (KB) version of the time-dependent Green's function method, a new *Ansatz* for the approximation of a spectral function is offered. The *Ansatz* possesses all the advantages of quasiparticle and extended quasiparticle approximations and satisfies the KB equation for a spectral function in the case of slightly nonequilibrium system when disturbances in space and time are taken into consideration in the gradient approximation. This feature opens opportunities for the microscopic derivation of the Landau kinetic equation for the quasiparticle distribution function of the normal Fermi liquid and provides the widening of these equation's temperature range of validity.

keywords: Spectral function, normal Fermi liquid, quasiparticle distribution, density matrix.
PACS number(s): 21.65.+f, 72.10.Bg, 72.20Dp.

1. INTRODUCTION

The only microscopic theory which is capable of describing both the statistics and dynamics in a comprehensive way is the Green's function approach developed in different varieties for equilibrium and nonequilibrium problems and for zero and finite temperatures. The initial emphasis in application of Green's function techniques was to understand the properties of normal condensed matter systems, superconductors, and superfluids.

Microscopic models used for a description of quantum interacting many-body systems involve spectral functions which play a central role in the formalism. Spectral functions are fundamental in describing the nuclear correlations, electrons correlations in metals and semiconductors, and in many other equilibrium and nonequilibrium properties. The most general approach to the problem is based on the real-time Green's function formalism of Martin and Schwinger, further developed by Kadanoff and Baym [1]. Keldysh [2] developed a diagram version of the theory for nonequilibrium systems equivalent to the Kadanoff-Baym (KB) approach. The KB equations were used in several contributions to this evolving field with applications to nuclear matter [3], to one- and two-band semiconductors [4, 5], to electron plasmas [6]–[8], etc. The various approximations of the KB equations differ essentially by the reduction schemes of the two-time Green's functions to the reduced equilibrium density matrix and to the quasiparticle distributions. There are several papers on spectral functions published during the past years using different methods and approximations [9]–[13]. The nonequilibrium extension of the KB formalism has been recovered within the quasiparticle approach to kinetic equation for weakly interacting particles and referred to as a modified KB *Ansatz* [22]–[24].

In a variety of works beginning with [1], the spectral function was approximated by a delta function of energy peaked at the quasiparticle energy. Such a simple approximation falls short of describing many important features of the systems under consideration. Improved spectral functions were necessary and some variants were offered in [11, 12, 23]. The extended quasiparticle (EQP) approximation was introduced and it was shown to remedy

some of the faults of the simpler quasiparticle (QP) approximation. Thus the EQP approximation was a considerable improvement of the formalism and it was used to compare the zero-temperature case of the KB approach with Brueckner theory of nuclear matter [11, 12]. The EQP approximation can be modified slightly while maintaining its simplicity and renormalization property. One possible modification of EQP was offered in [12], but it was not even mentioned among possible forms of spectral functions in [13].

Further development of the KB theory was performed in several different directions. Numerical solutions of the KB equations were presented in [14]. The extended quasiparticle picture which was first offered in [15], was developed for small scattering rates in [16]–[18]. The separation of pole and off-pole parts of a spectral function was discussed in detail in [17]. The nonlocal quasiparticle kinetic equation for the momentum-, space-, and time-dependent distribution function, which has the form of a Boltzmann equation with the quasiparticle energy, was derived in [18]. The generalization of the KB equations to the case of arbitrary initial correlation was performed in [19]. Correlation effects related to the collision integral were shown to cause a damping of the spectral function [20]. However, for the present analysis (see below), like in some other cases [21], a detailed self-consistent microscopic treatment of the correlation effects on the spectral function can be avoided.

Besides other applications, the spectral function was used for the derivation of the kinetic equation of the phenomenological Landau normal Fermi liquid theory. The initial derivation of this equation was produced by Kadanoff and Baym on the basis of the quasiparticle approximation for the spectral function and was continued by some followers who used the extended quasiparticle approximation. In both cases, the second Poisson bracket in the right side of the KB generalized kinetic equation could not be eliminated in a lawful mathematical way. This was the reason of the narrowing of the temperature range of validity of Landau's equation.

This paper is devoted to the explanation of the successfulness of the phenomenological Landau theory of normal Fermi liquid and its applicability far beyond its temperature range of validity established on the basis of quasiparticle and extended quasiparticle *Ansätze* for the spectral function. We offer another approximation for the spectral function which possesses the advantage typical for QP and EQP approximations but has an additional property to satisfy exactly the KB equation for the spectral function in the case of slightly nonequilibrium systems. It makes this form of the spectral function preferable in the nonequilibrium systems and opens certain opportunities for the widening of the range of validity of Landau Fermi-liquid kinetic equation.

The following section contains a concise presentation of the necessary formulas of the KB theory for both equilibrium and slightly nonequilibrium systems which will be used in Sec. 3 for a comparison of three different approximations for the spectral function. Section 4 contains the analysis of the question that what approximation for a spectral functions suits better to the strict results of Kadanoff and Baym for the nonequilibrium case and deals with the Landau kinetic equation for the quasiparticles in the normal Fermi liquid which is proved to be valid in a wider temperature region than it was considered in the initial derivation on the basis of the KB formalism. In Sec. 5 a discussion and summary are presented.

2. MAIN FORMULAS OF THE KB FORMALISM

We will introduce the main results of the KB theory for fermion systems [1] keeping in mind the questions which will be discussed below. The KB formalism leads to the following

general expression for the one-particle spectral function $a(\vec{p}\omega)$ of a system in equilibrium:

$$(1) \quad a(\vec{p}\omega) = \frac{\Gamma(\vec{p}\omega)}{[\omega - E^{HF}(\vec{p}) - \text{Re } \sigma_c(\vec{p}\omega)]^2 + \frac{\Gamma^2(\vec{p}\omega)}{4}},$$

where $E^{HF}(\vec{p})$ is a one-particle energy in the Hartree-Fock approximation. Real and imaginary (Γ) parts of the correlation self-energy function σ_c are related to each other through the Hilbert transform,

$$(2) \quad \text{Re } \sigma_c(\vec{p}\omega) = P \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\Gamma(\vec{p}\omega')}{\omega - \omega'}.$$

Here, P refers to a principal value integration.

The spectral function satisfies the exact sum rule:

$$(3) \quad \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} a(\vec{p}\omega) = 1,$$

for all the values of \vec{p} . This result follows directly from the commutator relations for field operators and can serve as a keystone for the checking of the validity of all approximations for the spectral function (1).

In the Hartree-Fock approximation, when the correlation self-energy function $\text{Re } \sigma_c(\vec{p}\omega) = 0$, the spectral function turns to be a delta function of the Hartree-Fock energy:

$$(4) \quad a_{HF}(\vec{p}\omega) = 2\pi\delta[\omega - E^{HF}(\vec{p})],$$

and the sum rule (3) is trivially satisfied identically for any value of $\sigma^{HF}(\vec{p})$. The simplest approximation in the case when $\text{Re } \sigma_c(\vec{p}\omega) \neq 0$ is the so called (QP) approximation:

$$(5) \quad a_{QP} = 2\pi Z(\vec{p})\delta(\omega - E(\vec{p})),$$

where $E(\vec{p})$ is the solution of the equation:

$$(6) \quad E(\vec{p}) = E^{HF}(\vec{p}) + \text{Re } \sigma_c[\vec{p}, E(\vec{p})],$$

and the renormalizing factor $Z(\vec{p})$ is given by the expression

$$(7) \quad Z^{-1}(\vec{p}) = 1 - \left. \frac{\partial \text{Re } \sigma_c(\vec{p}\omega)}{\partial \omega} \right|_{\omega=E(\vec{p})}.$$

In the QP approximation, the sum rule (3) reads

$$(8) \quad \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} a_{QP}(\vec{p}\omega) = Z(\vec{p}).$$

A severe drawback with a_{QP} is that it normalizes to $Z(\vec{p})$ rather than to 1 as in Eq. (3). We will see below how this deficiency is removed by the extended quasiparticle approximation [11]–[13] and by other more advanced approximations.

In the case of slowly varying in space and time disturbances, after the transition to Wigner coordinates,

$$(9) \quad \vec{R} = \frac{1}{2}(\vec{r}_1 + \vec{r}_{1'}), \quad \vec{r} = \vec{r}_1 - \vec{r}_{1'}, \quad T = \frac{1}{2}(t_1 + t_{1'}), \quad t = t_1 - t_{1'},$$

and the performance of the Fourier transform with respect to \vec{r}, t , all the quantities entering the theory are considered to be the functions of $\vec{p}, \omega, \vec{R}, T$, for example,

$$a = a(\vec{p}\omega; \vec{R}T).$$

If we take into account only the first derivatives with respect to slowly varying quantities \vec{R} and T in the KB equations for the correlation functions, we come to the following equation for the spectral function $a(\vec{p}\omega; \vec{R}T)$ [1]:

$$(10) \quad [\omega - E^{HF}(\vec{p}; \vec{R}T) - \text{Re } \sigma_c(\vec{p}\omega; \vec{R}T), a(\vec{p}\omega; \vec{R}T)] \\ + [\text{Re } g(\vec{p}\omega; \vec{R}T), \Gamma(\vec{p}\omega; \vec{R}T)] = 0,$$

and to the generalized KB kinetic equation for the correlation function $g^<(\vec{p}\omega; \vec{R}T)$:

$$(11) \quad [\omega - E^{HF}(\vec{p}; \vec{R}T) - \text{Re } \sigma_c(\vec{p}\omega; \vec{R}T), g^<(\vec{p}\omega; \vec{R}T)] \\ + [\text{Re } g(\vec{p}\omega; \vec{R}T), \sigma^<(\vec{p}\omega; \vec{R}T)] = (\sigma^<g^> - \sigma^>g^<)(\vec{p}\omega; \vec{R}T).$$

Here, $[A, B]$ is the so called generalized Poisson bracket, defined by the expression:

$$(12) \quad [A, B] = \frac{\partial A}{\partial \omega} \frac{\partial B}{\partial T} - \frac{\partial A}{\partial T} \frac{\partial B}{\partial \omega} - \nabla_{\vec{p}} A \cdot \nabla_{\vec{R}} B + \nabla_{\vec{R}} A \cdot \nabla_{\vec{p}} B,$$

E^{HF} and $\text{Re } \sigma_c$ include the interaction with the external field $U(\vec{R}T)$. The exact solution of Eq. (10) is given by the expression

$$(13) \quad g(\vec{p}z; \vec{R}T) = [z - E^{HF}(\vec{p}; \vec{R}T) - \text{Re } \sigma_c(\vec{p}z; \vec{R}T)]^{-1},$$

where all the functions, such as $a(\vec{p}\omega; \vec{R}T)$, $\text{Re } g(\vec{p}\omega; \vec{R}T)$, etc., are determined by the same formulas as in the equilibrium case with all entering quantities depending on $\vec{p}, \omega, \vec{R}, T$. For example,

$$\text{Re } \sigma_c(\vec{p}\omega; \vec{R}T) = P \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\Gamma(\vec{p}\omega'; \vec{R}T)}{\omega - \omega'}.$$

In fact, the solution (13) gives almost the same evaluation of the spectral function a as in the equilibrium case:

$$a(\vec{p}\omega; \vec{R}T) = \frac{\Gamma(\vec{p}\omega; \vec{R}T)}{[\omega - E^{HF}(\vec{p}; \vec{R}T) - \text{Re } \sigma_c(\vec{p}\omega; \vec{R}T)]^2 + \frac{\Gamma^2(\vec{p}\omega; \vec{R}T)}{4}}.$$

Since the solution (13) is of exactly the same form as the equilibrium solution, it must reduce to the equilibrium solution as $T \rightarrow -\infty$. Thus, it satisfies the initial condition on the equation of motion. This result means that the approximation for the nonequilibrium spectral function can be written in the same form as in the equilibrium case.

Equation (11) provides an exact description of the response to slowly varying disturbance. All the quantities appearing in this equation may be expressed in terms of the correlation functions $g^<$ and $g^>$. In particular, $\sigma^<$ and $\sigma^>$ are defined by a Green's function approximation that gives the self-energy in terms of $g^<$ and $g^>$.

3. APPROXIMATIONS FOR THE SPECTRAL FUNCTION

We will consider three different approximations for the spectral function which we will call *Ansatz 1*, *Ansatz 2*, and *Ansatz 3* correspondingly. Thus, we have

$$(14) \quad \text{Ansatz 1 :} \quad a_{EQP} = 2\pi Z(\vec{p})\delta(\omega - E) + P \frac{\Gamma(\omega)}{(\omega - E)^2};$$

$$(15) \quad \text{Ansatz 2 :} \quad a_{iQP} = 2\pi Z(\vec{p})\delta(\omega - E) + Z(\vec{p})P \frac{\Gamma(\omega)}{(\omega - E)^2};$$

$$(16) \quad \text{Ansatz 3 :} \quad a_i = 2\pi Z(\vec{p})\delta(\omega - E) + Z^2(\vec{p})P \frac{\Gamma(\omega)}{(\omega - E)^2}.$$

Here, E is determined by Eq. (6).

The *Ansatz* 1 corresponds to the extended quasiparticle approximation which was introduced, discussed, and used for numerical calculation in [11]–[13]. The first form of this *Ansatz* was offered in [25]. The *Ansatz* 2 as it was already mentioned above, offered in [12], but not discussed and was never used for the numerical calculation and was not even mentioned in [13]. A little bit different in technical details but the same in principle way of introducing the *Ansatz* 1 was offered in [23]. We would like to emphasize that the forms (14) and (15) of the spectral function were constructed in [11]–[13], [23] on the basis of the general expression (1) and were obtained by means of the Taylor expansion in powers of Γ in the frame of different approximations. In reality, there does not exist a mathematically strict correct form for the expansion of (1) in power series of $\Gamma(\omega)$ which starts with the delta function when $\Gamma(\omega) = 0$. Thus, expression (14) and (15) should be considered as some true-like approximate forms, as it is recognized in [23], which have better qualities compared with the quasiparticle *Ansatz* (5). In particular, Eqs. (14) and (15) obey the sum rule (3) (see below). Neither can be obtained in a strict way the improved *Ansatz* 3 a_i offered by us. The origin of the possibility of such *Ansatz* on equal terms with expressions (14) and (15) can be shown on the basis of the following consideration.

We start with a well known relation of the Fourier transform in the case of a constant value of c :

$$(17) \quad \int_{-\infty}^{\infty} e^{-|t|c} e^{itx} dt = \frac{2c}{c^2 + x^2}, \quad c > 0.$$

Expanding the first exponent in the left side of Eq. (17) in Taylor series, we get

$$(18) \quad \frac{2c}{c^2 + x^2} = \int_{-\infty}^{\infty} \left(1 - c|t| + \frac{c^2 t^2}{2!} - \dots \right) e^{itx} dt.$$

Now we use the formulas equivalent to those represented in [26]–[28]:

$$(19) \quad \int_{-\infty}^{\infty} t^{(2n)} e^{itx} dt = 2\pi(-i)^{2n} \delta^{(2n)}(x), \quad n = 0, 1, 2, \dots$$

$$(20) \quad \int_{-\infty}^{\infty} |t|^{(2n+1)} e^{itx} dt = -2 \sin \left((2n+1) \frac{\pi}{2} \right) (2n+1)! \frac{1}{|x|^{2n+2}},$$

$$n = 0, 1, 2, \dots$$

If the quantity $\Gamma(\omega)$ in Eq. (1) is constant, then the expression (18) would lead to a strict correct expansion of spectral function $a(\omega)$ in terms of the power series of Γ . However, $\Gamma(\omega)$ cannot be constant due to the dispersion relation (2). In the case of $\Gamma(\omega)$ depending on ω , one can rely only on the first two terms of the expansion: the delta function independent of $\Gamma(\omega)$ and the term proportional to $\Gamma(\omega)$. Then one should notice that the quantity x in Eq. (17) or (18) corresponds to $\omega - E^{HF}(\vec{p}) - \text{Re } \sigma_c(\vec{p}\omega)$ in the formula (1). Taking into account the relation (6), it becomes clear with the precision to the second derivative $\frac{\partial^2 \text{Re } \sigma_c}{\partial \omega^2}$, the term of the expansion (18) proportional to $\Gamma(\omega)$ involves $Z^2(p)$ in the numerator, and we come to the formula (16). Indeed, in the mentioned approximation we have

$$\omega - E^{HF}(\vec{p}) - \text{Re } \sigma_c(\vec{p}\omega) = Z^{-1}(\vec{p})[\omega - E(\vec{p})].$$

Now we should compare the expressions (14)–(16) on the basis of strict results obtained in the KB theory. We will check the correspondence of these expressions to the sum rule (3), the second sum rule on the basis of energy considerations and to Eq. (10) which determines the spectral function in the case of slowly varying in space and time disturbances. We start

with the sum rule (3). Substitute *Ansatz* 1 given by Eq. (14) to Eq. (3) and we get

$$(21) \quad \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} a_{EQP} = Z(\vec{p}) + P \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Gamma(\omega)}{(\omega - E)^2}.$$

Due to the dispersion relation (2) we have

$$(22) \quad \frac{\partial \operatorname{Re} \sigma_c(\omega)}{\partial \omega} \Big|_{E=} = -P \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Gamma(\omega)}{(\omega - E)^2},$$

and Eq. (21) becomes

$$(23) \quad \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} a_{EQP} = \frac{1}{1 - \frac{\partial \operatorname{Re} \sigma_c(\omega)}{\partial \omega} \Big|_E} - \frac{\partial \operatorname{Re} \sigma_c(\omega)}{\partial \omega} \Big|_{E \approx 1},$$

with the precision to $(\frac{\partial \operatorname{Re} \sigma_c}{\partial \omega})^2$.

The *Ansatz* 2 gives

$$(24) \quad \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} a_{iQP} = Z(\vec{p}) \left(1 + P \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Gamma(\omega)}{(\omega - E)^2} \right).$$

Taking into account the relation (22), we see that in the case under consideration the integral (24) is equal to 1.

Substituting Eq. (16) into Eq. (3) and taking into account (22), one gets

$$(25) \quad \int \frac{d\omega}{2\pi} a_i = Z(\vec{p}) \left(1 - Z \frac{\partial \operatorname{Re} \sigma_c(\omega)}{\partial \omega} \Big|_E \right) \approx 1,$$

with the precision to $(\frac{\partial \operatorname{Re} \sigma_c(\omega)}{\partial \omega})^2$.

Thus, formally the *Ansatz* 2 is the best in the sense of sum rule (3), but errors brought by *Ansatz* 1 and *Ansatz* 3 can be neglected in the approximation under consideration.

We can check the expressions (14)-(16) also with the help of the second sum rule [13]:

$$(26) \quad \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega a(\vec{p}\omega) = E^{HF}(\vec{p}).$$

The first terms in the right side of Eqs. (14)-(16) give the same result

$$(27) \quad \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega 2\pi Z(\vec{p}) \delta(\omega - E) = Z[E^{HF} + \operatorname{Re} \sigma_c(E)].$$

The contributions of the second terms in right sides of Eqs. (14)-(16) differ only in factors depending on the power of the renormalizing factor Z. The corresponding integral in each case is:

$$(28) \quad \begin{aligned} P \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\omega \Gamma(\omega)}{(\omega - E)^2} &= P \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Gamma(\omega)}{(\omega - E)} + EP \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Gamma(\omega)}{(\omega - E)^2} \\ &= Z^{-1}[(1 - Z)E^{HF} - (2Z - 1)\operatorname{Re} \sigma_c(E)]. \end{aligned}$$

The last equality in Eq. (28) follows with the help of the expressions (2), (7), and (22). Now taking into account the renormalizing factors standing with the second terms in Eqs. (14)-(16), we get with the help of Eqs. (27) and (28) that the second sum rule (26) is valid for all the *ansätze* (14)-(16) with the same precision up to the terms of the order $(\frac{\partial \operatorname{Re} \sigma_c(\omega)}{\partial \omega})^2$ and in this sense they are equivalent in principle. However, they turn to be not equivalent in the sense of satisfying the Eqs. (10) and (11), although it is mentioned in [23] that the corresponding off-pole part of $g^<$ for the *Ansatz* 1 (14) compensates a "dominant part of the puzzling term $[\operatorname{Re} g, \sigma^<]$ " in the generalized kinetic equation (11). However,

represented in [23] analysis did not lead to mathematically lawful complete elimination of this puzzling term. In the next section, we show that the *Ansatz* 3 (16) solves the problem and consequently is the best in the nonequilibrium case.

4. KINETIC EQUATION OF THE LANDAU THEORY OF THE NORMAL FERMI LIQUID

According to the strict result (13), every approximate *Ansatz* for the spectral function should keep valid in the case of slowly varying in space and time disturbances. The only difference occurs due to the dependence of all quantities on \vec{R} and T . When we substitute Eqs. (14)-(16) to Eq. (10), the first terms in the right side of these expressions give

$$\begin{aligned} & [\omega - E^{HF}(\vec{p}) - \text{Re } \sigma_c(\vec{p}\omega; \vec{R}T), 2\pi Z(\vec{p})\delta(\omega - E)] \\ (29) \quad & = 2\pi Z(\vec{p}) \left[(\omega - E) \left(1 - \frac{\partial \text{Re } \sigma_c}{\partial \omega} \Big|_E \right), \delta(\omega - E) \right] = 0, \end{aligned}$$

due to the property of the generalized Poisson bracket

$$(30) \quad [A, f(A)] = 0.$$

The second term in the right side of Eq. (14) leads to the expression

$$\begin{aligned} & \left[\omega - E^{HF}(\vec{p}) - \text{Re } \sigma_c(\vec{p}\omega; \vec{R}T), \frac{\Gamma(\vec{p}\omega; \vec{R}T)}{(\omega - E)^2} \right] \\ (31) \quad & = Z^{-1}(\vec{p}) \frac{1}{(\omega - E)^2} [\omega - E, \Gamma], \end{aligned}$$

with the precision to $\frac{\partial^2 \text{Re } \sigma_c(\omega)}{\partial \omega^2}$. The second Poisson bracket in Eq. (10) due to Eq. (13) gives with the same precision the expression

$$\begin{aligned} & [\text{Re } g(\vec{p}\omega; \vec{R}T), \Gamma(\vec{p}\omega; \vec{R}T)] = Z(\vec{p}) \left[\frac{1}{(\omega - E)}, \Gamma \right] \\ (32) \quad & = -Z(\vec{p}) \frac{1}{(\omega - E)^2} [\omega - E, \Gamma]. \end{aligned}$$

Finally, collecting all the terms, we get that the left side of Eq. (10) in the case under consideration is different for Eqs. (14)-(16) and equals to

$$(33) \quad \textit{Ansatz 1} : \left[\frac{1}{(\omega - E)}, \Gamma \right] [Z(\vec{p}) - Z^{-1}(\vec{p})];$$

$$(34) \quad \textit{Ansatz 2} : \left[\frac{1}{(\omega - E)}, \Gamma \right] [Z(\vec{p}) - 1];$$

$$\textit{Ansatz 3} : \left[\frac{1}{(\omega - E)}, \Gamma \right] [Z(\vec{p}) - Z(\vec{p})] = 0.$$

The *Ansatz* 3 satisfies Eq. (10) exactly. It means that this *Ansatz* is preferable for considering slightly nonequilibrium systems when disturbances slowly vary in space and time. In particular, it turns out that the *Ansatz* 3 (16) opens opportunities for the derivation of the Landau Fermi liquid's theory kinetic equation for the quasiparticle distribution function. We will see that the usual considerations about the smallness of the quantity $\Gamma(\omega)$ are not necessary anymore and the puzzling term $[\text{Re } g, \sigma^<]$ in Eq. (11) is canceled completely by the off-pole part of the approximation (16).

From the pioneer work of Kadanoff and Baym, the derivation of the kinetic equation of normal Fermi liquid theory was based on the assumption of the smallness of the functions

$\sigma^<$ and $\sigma^>$ near the Fermi level μ of the system [1, 29]. Indeed, defining a local occupation number $f(\vec{p}\omega; \vec{R}T)$ by writing

$$(35) \quad g^<(\vec{p}\omega; \vec{R}T) = a(\vec{p}\omega; \vec{R}T)f(\vec{p}\omega; \vec{R}T),$$

where in equilibrium, at zero temperature

$$(36) \quad f(\vec{p}\omega; \vec{R}T) \rightarrow f(\omega) = \begin{cases} 0 & \text{for } \omega > \mu \\ 1 & \text{for } \omega < \mu \end{cases},$$

one comes to the relations

$$(37) \quad \begin{aligned} \sigma^>(\vec{p}\omega) &= 0 & \text{for } \omega < \mu &, \\ \sigma^<(\vec{p}\omega) &= 0 & \text{for } \omega > \mu &. \end{aligned}$$

The proof of these relations depends only the fact that $f = 1$ for $\omega < \mu$ and $f = 0$ for $\omega > \mu$. Since we are assuming that f has a similar behavior in the slightly nonequilibrium case, it follows that

$$(38) \quad \begin{aligned} \sigma^>(\vec{p}\omega; \vec{R}T) &= 0 & \text{for } \omega < \mu(\vec{R}T) &, \\ \sigma^<(\vec{p}\omega; \vec{R}T) &= 0 & \text{for } \omega > \mu(\vec{R}T) &, \end{aligned}$$

where $\mu(\vec{R}T)$ is a local chemical potential of the system. For normal Fermi systems, $\sigma^>$ and $\sigma^<$ are continuous functions at $\omega = \mu$. It means that $\Gamma = \sigma^> + \sigma^<$ is small near μ . As it was shown in [1], these assumptions about f for ω appreciably greater or less than $\mu(\vec{R}T)$ lead to a consistent solution of the generalized kinetic equation (11). In such a situation when $\sigma^>$ and $\sigma^<$ are both negligible for ω near μ , Eq. (11) in collisionless approximation can be written in the form

$$(39) \quad [\omega - E^{HF}(\vec{p}; \vec{R}T) - \text{Re } \sigma_c(\vec{p}; \vec{R}T), a(\vec{p}\omega; \vec{R}T)f(\vec{p}\omega; \vec{R}T)] = 0.$$

The second Poisson bracket in the right side of Eq. (11) is dropped out due to the discussed considerations about the smallness of Γ near μ .

Using the quasiparticle *Ansatz* (5) for the spectral function a , one comes to the kinetic equation for the quasiparticle distribution function $n(\vec{p}, \vec{R}T)$ of the normal Fermi liquid theory [1]:

$$(40) \quad \frac{\partial n}{\partial T} + \nabla_{\vec{p}} E \cdot \nabla_{\vec{R}} n - \nabla_{\vec{R}} E \cdot \nabla_{\vec{p}} n = 0,$$

$$n(\vec{p}; \vec{R}T) = f(\vec{p}\omega; \vec{R}T) |_{\omega=E(\vec{p}; \vec{R}T)}.$$

The smallness of the function Γ is a keystone in the above considerations and such a smallness looks like a necessary condition of the validity of Eq. (40), which is supposed to be valid only in the region $\omega \approx \mu(\vec{R}T)$. However, it can be shown that neglecting collisions, Eq. (40) stays valid with the precision to Γ^2 if one uses *Ansatz* 3 (16) for the spectral function a instead of the quasiparticle *Ansatz* (5). In this case, the collisionless equation (11) is written in the form

$$(41) \quad [\omega - E^{HF} - \text{Re } \sigma_c, af] + [\text{Re } g, \sigma^<] = 0.$$

The first term in all the *Ansätze* (14)-(16) which coincides with the quasiparticle *Ansatz* (5) leads directly to Eq. (40) as it is shown in [1]. The second Poisson bracket in Eq. (41)

in the linear approximation in Γ gives the result

$$(42) \quad [\text{Re } g, \sigma^<] = \left[\frac{1}{\omega - E^{HF} - \text{Re } \sigma_c}, \Gamma f \right] = Z \left[\frac{1}{\omega - E}, \Gamma f \right] \\ = Z[\Gamma f, \omega - E] \frac{1}{(\omega - E)^2}.$$

It is easy to see that this term will be compensated by the second term in the *Ansatz* 3. Indeed, substituting this second term to the first Poisson bracket in Eq. (41), one gets

$$(43) \quad Z^2 \left[\omega - E^{HF} - \text{Re } \sigma_c, \frac{\Gamma f}{(\omega - E)^2} \right] = Z[\omega - E, \Gamma f] \frac{1}{(\omega - E)^2},$$

and this term compensates Eq. (42) in Eq. (41). Observe that neither *Ansatz* 1 nor *Ansatz* 2 will lead to this result.

We would like to stress that all considerations based on the relations (36)–(38) were necessary only for the sake of the elimination of the second Poisson bracket in Eq. (41). The usage of the *Ansatz* 3 (16) makes the smallness of the function Γ not necessary. The quantity $\Gamma(\omega)$ can be finite. For qualitative estimation of the precision, we can use the third term in the expansion (18) which corresponds to $\Gamma^2(\omega)$. Substituting the term proportional to $\Gamma^2(\omega)$ into Eqs. (10) and (11), it is not difficult to show that in the gradient approximation these equations are valid up to the terms of the order $\Gamma^2(\omega)$.

This result means that the collective excitation spectrum of the system which is determined by the kinetic equation (40) is not changed when we take into considerations the term in the spectral function proportional to Γ . At the same time, physical quantities which are determined by the correlation function $g^<$ will obtain additional terms due to the second term in the expression for the spectral function. In particular, the reduced density matrix,

$$(44) \quad \rho(\vec{p}; \vec{R}T) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} g^<(\vec{p}\omega; \vec{R}T),$$

will possess power tails while the quasiparticle distribution n decrease exponentially with energy.

5. DISCUSSION AND SUMMARY

The differences in the form of the expressions (14)–(16) come through the ways they were obtained. When the *Ansatz* for the spectral function is obtained by a Taylor expansion of Eq. (1) around the quasiparticle peak, it involves the necessity of the conditions $\Gamma \ll \text{Re } \sigma_c$ and $\frac{\partial \text{Re } \sigma_c}{\partial \omega} \ll 1$ [11]–[13]. The approach (18) based on the Fourier transform demands another condition $(\omega - E) \frac{\partial \ln \Gamma(\omega)}{\partial \omega} \ll 1$, which means that a relation between Γ and $\text{Re } \sigma_c$ can be arbitrary, in the frame of the Hilbert transform (2).

The possibility for Γ to be not small plays an important role for the increasing of the range of validity of the kinetic Fermi liquid equation. The comparison of the results obtained on the basis of this equation with experimental data confirmed the validity of the equation far beyond the limits established by the conditions (36)–(38) and, consequently, for a essentially larger temperature interval.

The *Ansatz* 3 (16) leads to the elimination of the second Poisson bracket in Eq. (11) in a very natural way. As it turned out, this term in the generalized KB kinetic equation (11) is the main obstacle on the way of the extension of Fermi liquid equation (40) to a larger temperature interval [23]. We should mention that the validity of the kinetic equation (40) in the case of finite values of Γ was shown by different mathematical method in [30] but the

result looked contradicting to the commonly accepted conditions (36)-(38). Only the direct “lawful” elimination of the term $[\text{Re } g, \sigma^<]$ in Eq. (11) makes the situation clear.

Despite the absence of the rigorous foundation of the validity of the Fermi-liquid equations in the case of large Γ , these equations were successfully used for the variety of systems of strongly interacting particles. Thus, such equations were used for theoretical description of the magneto-ordered state in 3d metals in the framework of the Anderson periodic model [31]. The value of Γ was considered to be constant and of the same order as other energy parameters of the system: the width of the d level was considered to be equal to a relatively large jump parameter v : $v \sim \Gamma \sim 1 \text{ eV}$. The obtained results turned to be in a good agreement with experimented data for the photoeffect on polarized electrons [32, 33]. In conclusion, we will mention that the *Ansatz* 3 (16) can be preferable also for calculations of the equilibrium properties such as produced for a nuclear matter in [12]. As it is stated in this work, using the extended quasiparticle approximation (14), the total binding energy is obtained to be

$$E_{EQP} = 17.4 \text{ MeV/nucleon.}$$

In contrast, the quasiparticle approximation (5) leads to the result

$$E_{QP} = 15.9 \text{ MeV/nucleon.}$$

Bruckner theory meanwhile gives

$$E_B = 16.7 \text{ MeV/nucleon.}$$

Due to the extra factor $Z^2(\vec{p})[Z(\vec{p}) < 1]$ in the second term of the *Ansatz* 3 compared with the *Ansatz* 1, it is clear that the result of this *Ansatz* must be between the values of E_{QP} and E_{EQP} . It will be closer to the value of E_B than E_{EQP} and closer to the value of E_B than E_{QP} .

REFERENCES

- [1] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962/ Perseus Books, Cambridge, MA, 1989).
- [2] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)].
- [3] H. S. Kohler, Phys. Rev. C **51**, 3232 (1995); Phys. Rev. E **53**, 3145 (1996).
- [4] R. Binder, H. S. Kohler, M. Bonitz, and N. Kwong, Phys. Rev. B **55**, 5110 (1997).
- [5] N. H. Kwong, M. Bonitz, R. Binder, and H. S. Kohler, Phys. Status Solidi B **206**, 197 (1988).
- [6] M. Bonitz, D. Kremp, D. C. Scott, R. Binder, W. D. Kraeft, and H. S. Kohler, J. Phys.: Condens. Matter **8**, 6057 (1996).
- [7] M. Bonitz, R. Binder, and H. S. Kohler, Contrib. Plasma Phys. **37**, 101 (1997).
- [8] H. S. Kohler, and R. Binder, Contrib. Plasma Phys. **37**, 167 (1997).
- [9] O. Benhar, A. Fabrocini, and S. Fantoni, Nucl. Phys. A **505**, 267 (1998).
- [10] F. de Jong, and R. Malfliet, Phys. Rev. C **44**, 998 (1991).
- [11] H. S. Kohler, Phys. Rev. C **46**, 1687 (1992).
- [12] H. S. Kohler and R. Malfliet, Phys. Rev. C **48**, 1034 (1993).
- [13] H. S. Kohler and K. Morawetz, Phys. Rev. C **64**, 024613 (2001).
- [14] N. Hang Kwong and M. Bonitz, Phys. Rev. Lett. **84**, 1768 (2000).
- [15] R. A. Craig, Ann. Phys. (N.Y.) **40**, 416 (1966).
- [16] V. Spicka, and P. Lipavsky, and K. Morawetz, Phys. Lett. A **240**, 160 (1998).
- [17] K. Morawetz, P. Lipavsky, and V. Spicka, Ann. Phys. (N.Y.) **294**, 134 (2001).
- [18] P. Lipavsky, K. Morawetz, and V. Spicka, Ann. Phys. (N.Y.) **26**, 1 (2001).
- [19] D. Semkat, D. Kremp, and M. Bonitz, Phys. Rev. E **59**, 1557 (1999).
- [20] M. Bonitz, D. Semkat, and H. Haug, Eur. Phys. J. B **9**, 309 (1999).
- [21] M. Bonitz and J. W. Dufty, Condens. Matter Phys. **7**, 488 (2004).
- [22] V. Spicka, and P. Lipavsky, Phys. Rev. Lett. **73**, 3439 (1994).
- [23] V. Spicka and P. Lipavsky, Phys. Rev. B **52**, 14615 (1995).
- [24] K. Morawetz and G. Roepke, Phys. Rev. E **51**, 4246 (1995).

- [25] W. D. Kraeft, D. Kremp, W. Ebeling, and G. Ropke, *Quantum Statistics of Charged Particle Systems* (Akademie, Berlin, 1986).
- [26] E. C. Titchmarsh, *Introduction to the theory of Fourier Integrals* Oxford (Clarendon, Oxford, 1975).
- [27] M. J. Lighthill, *Introduction to Fourier Analysis and Generalized Functions* (Cambridge University Press, Cambridge, 1960).
- [28] Yu. A. Brychkov and A. P. Prudnikov, *Integral Transform of Generalized Functions* (Gordon and Breach, New York, 1989).
- [29] A. S. Kondratyev, and A. E. Kutchma, *Lectures in the Quantum Liquids Theory* (Leningrad University, 1989).
- [30] A. S. Kondratyev and I. E. Liublinskaya, Herald of Leningrad State Univ. **25**, 74 (1985).
- [31] A. K. Kazansky, A. S. Kondratyev, and V. M. Uzdin, Sov. Phys. JETP, **94**, 274 (1988).
- [32] A. M. Turner, Y. J. Chang, and J. L. Erskine, Phys. Rev. Lett. **48**, 348 (1982).
- [33] A. M. Turner and J. L. Erskine, Phys. Rev. B **28**, 5628 (1983).

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